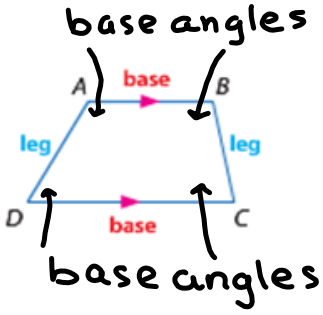
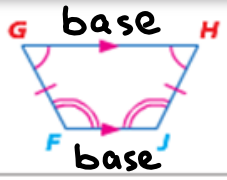
1 Properties of Trapezoids A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called **bases**. The nonparallel sides are called **legs**. The **base angles** are formed by the base and one of the legs. In trapezoid $ABCD$, $\angle A$ and $\angle B$ are one pair of base angles and $\angle C$ and $\angle D$ are the other pair. If the legs of a trapezoid are congruent, then it is an **isosceles trapezoid**.



Theorems Isosceles Trapezoids $2 \cong$ legs

6.21 If a trapezoid is isosceles, then each pair of base angles is congruent.

Example If trapezoid *FGHJ* is isosceles, then $\angle G \cong \angle H$ and $\angle F \cong \angle J$.

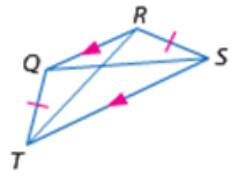


6.22 If a trapezoid has one pair of congruent base angles, then it is an isosceles trapezoid.

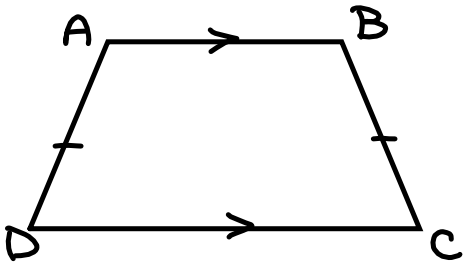
Example If $\angle L \cong \angle M$, then trapezoid *KLMP* is isosceles.

6.23 A trapezoid is isosceles if and only if its diagonals are congruent.

Example If trapezoid *QRST* is isosceles, then $\overline{QS} \cong \overline{RT}$. Likewise, if $\overline{QS} \cong \overline{RT}$, then trapezoid *QRST* is isosceles.



In an isosceles trapezoid, the opposite angles are supplementary.



base angles

$\angle A \cong \angle B$

$\angle D \cong \angle C$

$\overline{AB} \parallel \overline{DC}$

consecutive interior angles are supplementary

$m\angle A + m\angle D = 180$

$m\angle B + m\angle C = 180$

$\downarrow$

$m\angle A + m\angle C = 180$

$m\angle B + m\angle D = 180$

by substitution.

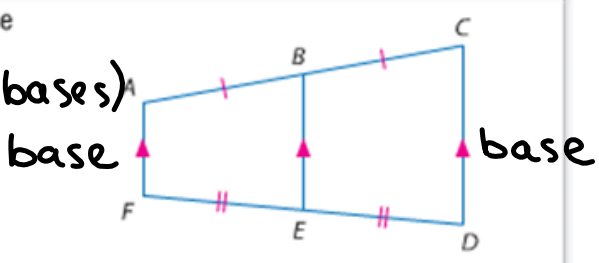
The **midsegment of a trapezoid** is the segment that connects the midpoints of the legs of the trapezoid.

The theorem below relates the midsegment and the bases of a trapezoid.

Theorem 6.24 Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the lengths of the bases. (average of length of bases)

Example If $\overline{BE}$ is the midsegment of trapezoid $ACDF$, then $\overline{AF} \parallel \overline{BE}$, $\overline{CD} \parallel \overline{BE}$, and $BE = \frac{1}{2}(AF + CD)$.

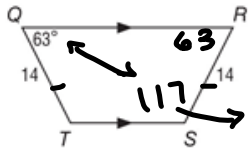


NOTE

A trapezoid midsegment is often referred to as "the median of a trapezoid."

TRAPEZOID EXAMPLES:

1. $m\angle S$ = 117

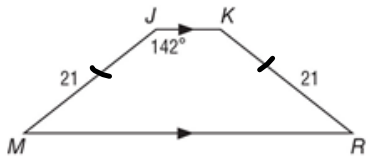


$m\angle R = 63$

$m\angle S =$ 180-63 = 117

Isosceles because $\overline{QT} \cong \overline{RS}$

2. $m\angle M$ 180-142= 38



Isosceles trapezoid because $\overline{JM} \cong \overline{KR}$

$\overline{TS}$ = midsegment

ALGEBRA For trapezoid *HJKL*, *T* and *S* are midpoints of the legs.

3. If *HJ* = 14 and *LK* = 42, find *TS*. $\overline{TS}=28$

$x=\frac{14+42}{2}=\frac{56}{2}=28$

4. If *LK* = 19 and *TS* = 15, find *HJ*. $HJ=11$

$15=\frac{x+19}{2} \rightarrow 30=x+19$ $-19\ \ -19$ $x=11$

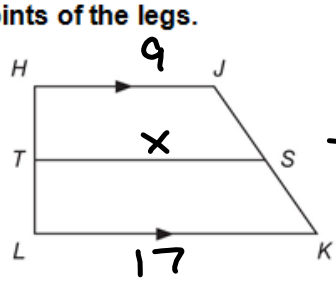
5. If *HJ* = 7 and *TS* = 10, find *LK*.

$\frac{10}{1}=\frac{7+x}{2}$ $20=7+x$ $-7\ \ -7$ $13=x=LK$

6. If *KL* = 17 and *JH* = 9, find *ST*.

$x=\frac{9+17}{2}=\frac{26}{2}=13$

$ST=13$



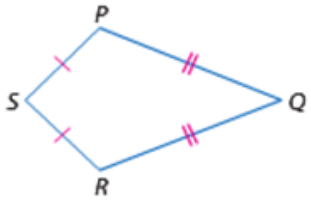
$\overline{HJ} \parallel \overline{LK}$

$\overline{HJ} \parallel \overline{TS} \parallel \overline{LK}$

$TS=\frac{HJ+LK}{2}$

formula

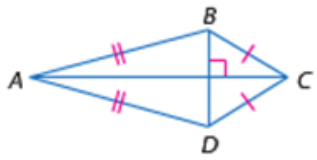
2 Properties of Kites A **kite** is a quadrilateral with exactly two pairs of consecutive congruent sides. Unlike a parallelogram, the opposite sides of a kite are not congruent or parallel.



Theorems Kites

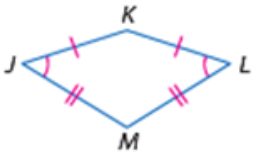
6.25 If a quadrilateral is a kite, then its diagonals are perpendicular.

Example If quadrilateral *ABCD* is a kite, then $\overline{AC} \perp \overline{BD}$.



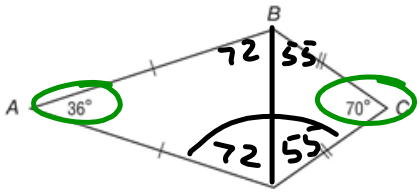
6.26 If a quadrilateral is a kite, then exactly one pair of opposite angles is congruent.

Example If quadrilateral *JKLM* is a kite, $\overline{JK} \cong \overline{KL}$, and $\overline{JM} \cong \overline{LM}$, then $\angle J \cong \angle L$ and $\angle K \ncong \angle M$.



KITE EXAMPLES:

9. $m\angle D$ $72+55=127$



$180-36=144$

$\frac{144}{2}=72$

$180-70=110$

$\frac{110}{2}=55$

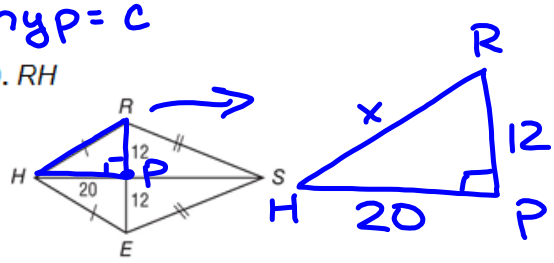
$360-(36+70)$

$360-106$

$\frac{254}{2}=127$

hyp = c

10. RH



$a^2+b^2=c^2$

$12^2+20^2=x^2$

$144+400=x^2$

$544=x^2$

$\sqrt{544}=\sqrt{x^2}$

$RH=\sqrt{544}$